A Sample Challenge
GO FIGURE 2000 Mathematical Challenge

Held Saturday, 4 Nov 2000

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Show your work. You can receive partial credit for partial solutions.
Please write all solutions clearly, and legibly.

The positive integers are the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 ,10, 11, 12, 13, 14, 15, 16, 17, 18,...

1. (a.) What is the smallest number among the five sums
   $1 + 36,\ 2 + 18,\ 3 + 12,\ 4 + 9,\ 6 + 6$?

   (b.) What is the smallest number among the six sums
   $1 + 72,\ 2 + 36,\ 3 + 24,\ 4 + 18,\ 6 + 12,\ 8 + 9$?

2. Find the sixth term *a* and the seventh term *b* of the geometric progression 18, 12, 8, 16/3, 32/9, *a*, *b*.
   [One multiplies a term by a fixed number to get the next term.]

3. Find integers *c* and *d*, each greater than 1, such that $2491 = c \times d$.
   [Hint: Find a pattern in $91 = 7 \times 13$, $391 = 17 \times 23$, $891 = 27 \times 33$, $1591 = 37 \times 43$.]

4. In an arithmetic progression, one adds a fixed number (which may be positive, negative, or zero) to any term to get the next term.
   (a) Find the 70th term of the unending arithmetic progression 5, 10, 15, 20...
   (b) Find the 70th term of the unending arithmetic progression 2, 7, 12, 17...
   [Advice: Compare terms with those in part (a).]
   (c) How many terms are there in the arithmetic progression 14, 19, 24,....., 994?
   (d) What is the average (that is, arithmetic mean) of the terms of the progression in part (c)?
   (e) Find the sum $14 + 19 + 24 + ... + 994$ of the terms of the progression in (c).

5. Powers b^e are defined for $e = 1, 2, 3, ...$ by $b^1 = b$, $b^2 = b \times b$, $b^3 = b \times b \times b$, $b^4 = b \times b \times b \times b$, and so on.
   (a.) Find an *r* in {0, 1, 2, 3} and a positive integer *q* such that $b^{123} = (b^4)^q \times b^r$.
   (b.) Find the units digit (that is, rightmost digit) of 7^{123}

6. Here each letter represents one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. For example, if $F = 4$ then $2F$ stands for 24 and $FF5$ stands for 445. Find digits *A*, *B*, and *C* such that $A2B \times 683 = 86BCA$.

7. (a) What is the smallest value of $f + g = h$ for positive integers *f*, *g*, and *h* such that $f \times g \times h = 792$?
   (b) What is the smallest sum of (more than three) positive integers whose product is 792?

PROBLEM 8 WILL BE USED ONLY TO BREAK TIES

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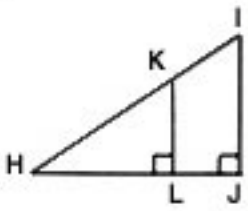
8. As clearly as you can, justify your answers for problem 7.

PROBLEMS 9, 10, 11 and 12 ARE OPTIONAL FOR STUDENTS IN GRADES 7, 8, 9.

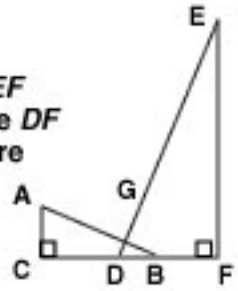
9. (a) Find the two numbers x that satisfy $(x - 50)^2 = 9$
   (b) Find the four numbers u that satisfy the equation $(u^2 - 17)^2 = 64$.

10. The point K is on the hypotenuse HI of the right triangle HIJ and L is on side HJ. Both angle HJI and angle HLK are right angles. The lengths of segments HL, LJ and HK are 112, 56, and 130 units, respectively. Find the lengths of segments KL and KI.



11. As in the figure, right triangles ABC and DEF overlap so that D is on side CB, B is on side DF and the hypotenuses meet at G. Lengths are as follows:

segments	AB	AC	CD	DB	BF	EF
length	13	5	8	4	6	24



(a) Find the area of ΔDBG.

(b) Find the length of segment AE.

12. There are 3 cars available to transport 3 girls and 5 boys on a field trip. Each car can hold up to 3 children. There are

$$3 \cdot \binom{8}{2} \cdot \binom{6}{3} \cdot \binom{3}{3} = 3 \left(\frac{8 \cdot 7}{2} \right) \cdot \left(\frac{6 \cdot 5 \cdot 4}{2 \cdot 3} \right) \cdot 1 = 1680$$

ways to assign the children to cars, since there are 3 choices for the car to hold only 2 children, $\binom{8}{2}$ ways to pick these 2 children, $\binom{6}{3}$ ways to assign 3 other children to another car, and $\binom{3}{3} = 1$ way to complete the assignment. How many of the 1680 ways assign 2 or 3 girls to one of the cars?

[The symbol $\binom{n}{k}$ stands for the number of distinct subsets with exactly k elements that can be chosen from a set of n elements. Also, $\binom{n}{0} = 1 = \binom{n}{n}$ and

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{1 \cdot 2 \cdot 3 \ldots r} \quad \text{for } r = 1, 2, \ldots, n-1.]$$

answers